On the Channel Quality of XL-MIMO Systems

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Contents

01 Background and Motivations

- Key Performance Indicators for **6G Systems**

To support emerging applications, the KPIs for 6G expected to provide an **order-of-magnitude** leap over 5G.

- ✓ Connection density: $10^6 \rightarrow 10^8$ devices/km^2 (**100** ×)
- ✓ Energy Efficiency: $10^7 \rightarrow 10^9$ bit/J (**100** ×)
- ✓ Reliability: 99.999% → 99.99999% (**100** ×)
- ✓ Peak Data Rate: 20 Gbps → 1 Tbps (**50** ×)
- ✓ Latency: 1ms → 0.1ms (**10** ×)

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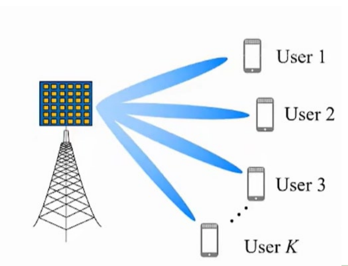
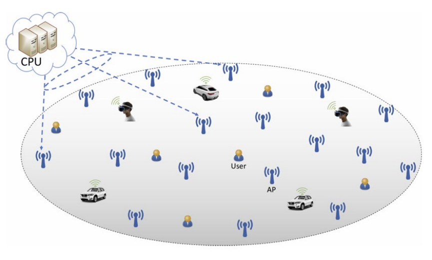
- Promising Candidate Technologies

Extremely Large-Scale MIMO



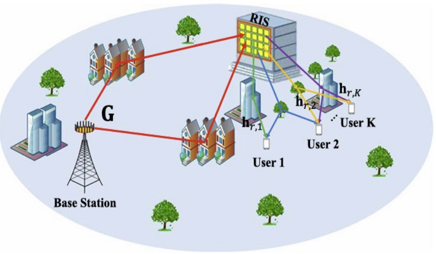
Spectrum Efficiency

Cell-Free MIMO



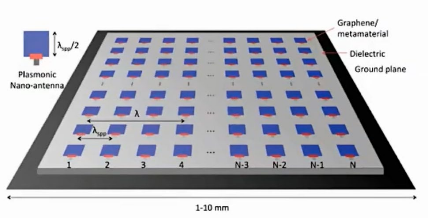
Distributed Cooperation

Reconfigurable Intelligent Surfaces



Coverage

Terahertz Communications



Peak Data Rate

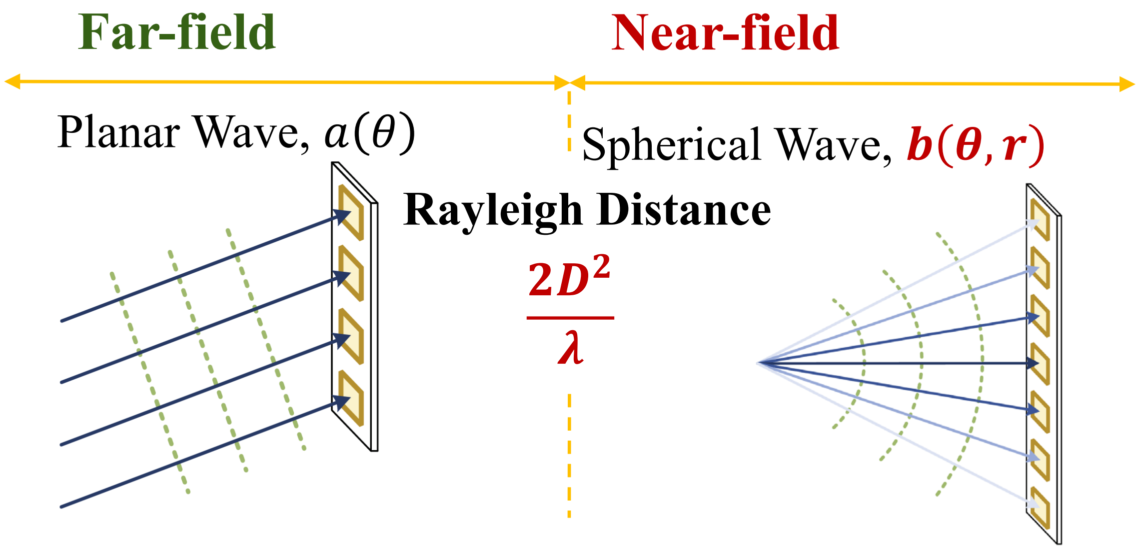
XL-MIMO is a Fundamental Technology for 6G Communications

01 Background and Motivations

Importance of XL-MIMO

- ☐ **Function**: support higher spectral efficiency, larger system capacity, lower latency, and more reliable transmission
- ☐ **Role**: a key pillar in 6G communication networks
- ☐ **New feature:** large antenna aperture pushes users into the **near-field region**

With **enlarged antenna apertures** and **shrinking cell sizes**, users are more likely to fall within the **near-field** region.



Far-field

Near-field

Planar Wave, $a(\theta)$

Spherical Wave, $\boldsymbol{b(\theta, r)}$

Rayleigh Distance

$$\frac{2D^2}{\lambda}$$

Far-Field Array Response Vector

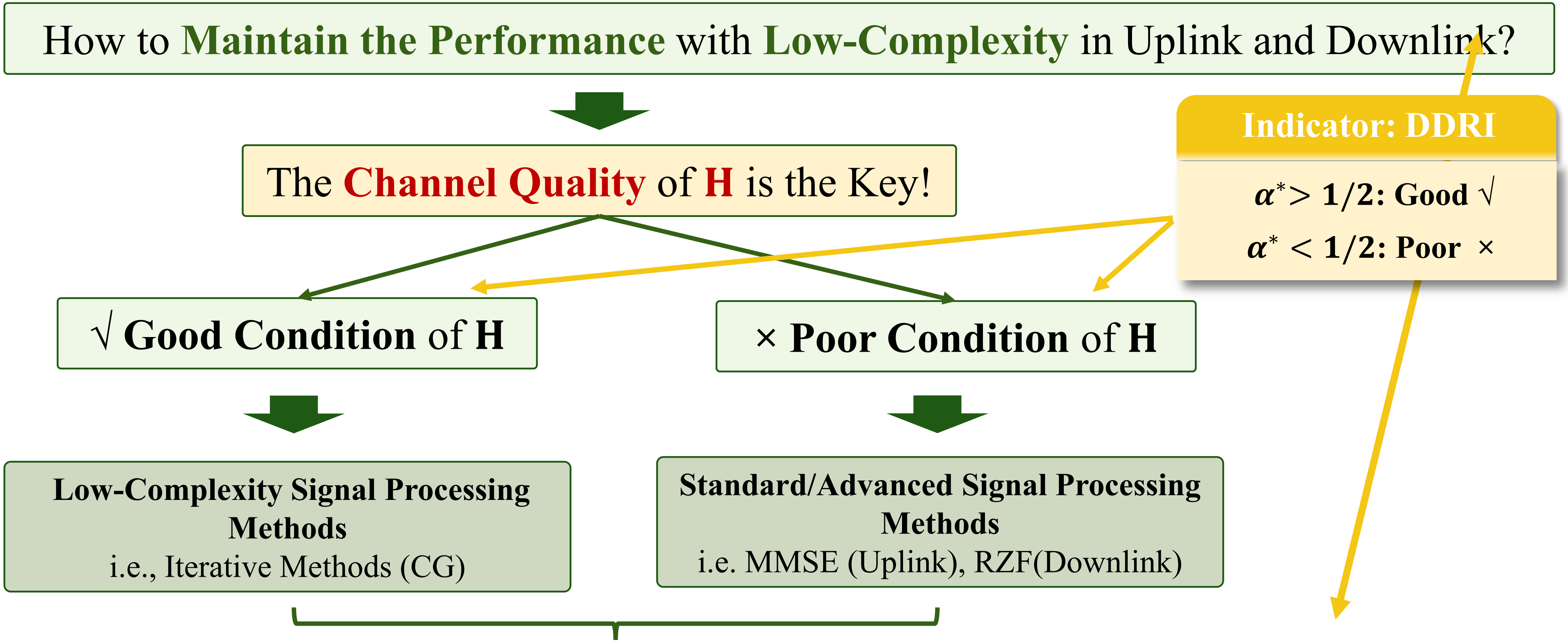
$$a(\theta) = \left[1, e^{-j2\pi\frac{d}{\lambda}sin\theta}, \dots, e^{-j2\pi(N-1)\frac{d}{\lambda}sin\theta}\right]^{\mathrm{T}}$$

Near-Field Array Response Vector

$$a(\theta) = \left[1, e^{-j2\pi\frac{d}{\lambda}sin\theta}, \dots, e^{-j2\pi(N-1)\frac{d}{\lambda}sin\theta}\right]^{\mathrm{T}}$$

The channel model of XL-MIMO becomes much more complicated than ever before

01 Background and Motivations



How to **Maintain the Performance** with **Low-Complexity** in Uplink and Downlink?

The **Channel Quality** of **H** is the Key!

Indicator: DDRI

$\boldsymbol{\alpha}^{*} >$ **1/2: Good** √

$\boldsymbol{\alpha}^{*} <$ **1/2: Poor** ×

√ **Good Condition** of **H**

× **Poor Condition** of **H**

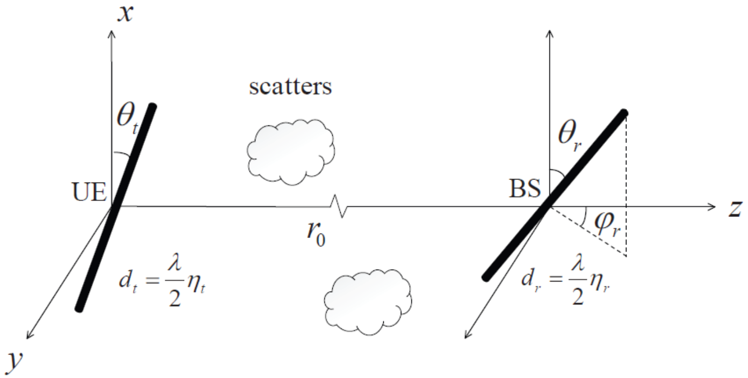
Low-Complexity Signal Processing Methods
i.e., Iterative Methods (CG)

Standard/Advanced Signal Processing Methods
i.e. MMSE (Uplink), RZF(Downlink)

How to Efficiently Estimate the Channel Quality is the Key!

02 System Model and Assumptions

Systems Configurations



- ☐ The BS and the UE are equipped with ULAs, containing N and K antenna elements, respectively.
- ☐ The **inter-element spacing** are d_t and d_r.
- ☐ The BS is centered at the origin and oriented by θ_r, while the UE center is located at distance r_0 with the **elevation angle** θ_t and **azimuth angle** φ_t.
- ☐ The exact distance between elements

$$\mathrm{r}_{\mathrm{n,k}} = \|\mathbf{b}_{\mathbf{n}} - \mathbf{t}_{\mathbf{k}}\|$$

Near-field geometry determines the structure and correlation of the Channel Matrix H

Near-Field Channel Matrix

- The near-field channel matrix is composed of both **LoS** and **NLoS** components[1].

$$\mathbf{H} = \sqrt{\frac{\kappa}{\kappa+1}}\,\mathbf{H}_{\mathrm{LoS}} + \sqrt{\frac{1}{\kappa+1}}\,\mathbf{H}_{\mathrm{NLoS}},$$

  - ☐ **LoS** components

$$[\mathbf{H}_{\mathrm{LoS}}]_{n,k} = \frac{\rho}{r_{n,k}} \exp\left(-\mathrm{j}\frac{2\pi}{\lambda} r_{n,k}\right)$$

  - ☐ **NLoS** components

$$[\mathbf{H}_{\mathrm{NLoS}}]_{n,k} \sim \frac{\rho}{r_{n,k}} \mathcal{CN}(0,1).$$

- **Assumptions**
  - ☐ **Amplitude**: amplitude variations across the array are negligible.

$$\frac{\rho}{r_{n,k}} \approx \frac{\rho}{r_0} \triangleq \tilde{\beta}$$

  - ☐ **Phase**: $\mathrm{r}_{\mathrm{n,k}}$ is approximated by **Fresnel expansion.**

$$\mathrm{r}_{\mathrm{n,k}} \approx r_0 + \left(k d_t \sin\theta_t \cos\varphi_t + \frac{(k d_t \sin\theta_t \sin\varphi_t)^2}{2r_0} + \frac{(k d_t \cos\theta_t)^2}{2r_0}\right) + \\ + \left(-n d_r \sin\theta_r + \frac{(n d_r \cos\theta_r)^2}{2r_0}\right) - \frac{d_t d_r n k \cos\theta_t \cos\theta_r}{r_0}$$

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[1] J. Liu, Y. Ma, and R. Tafazolli, “A spatially non-stationary fading channel model for simulation and (semi-) analytical study of ELAA-MIMO,” IEEE Trans. Wireless Commun., vol. 23, no. 5, pp. 5203–5218, 2023.

03 Channel Quality Indicator: DDRI

Challenge 1: How to Evaluate XL-MIMO Channel Quality Effectively?

Definition 1: Diagonal Dominance Ratio Indicator, DDRI

Given the Gram matrix $\mathbf{G}=\mathbf{H}^{\mathbf{H}}\mathbf{H}$, the diagonal ratio of the i-th stream is

$$\alpha_{i}=\frac{|g_{ii}|}{\sum_{j=1}^{K}|g_{ij}|}=\frac{|g_{ii}|}{|g_{ii}|+\sum_{j\neq i}^{K}|g_{ij}|}$$

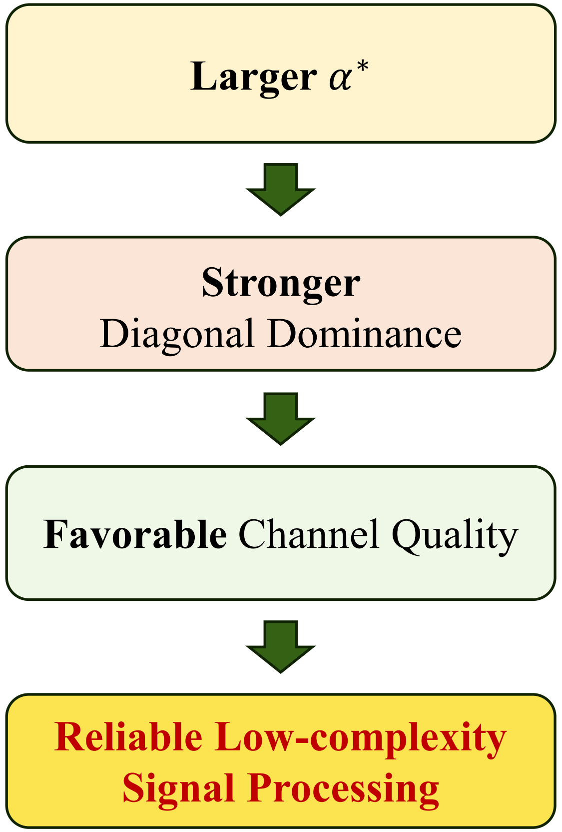
The proposed DDRI is defined as the worst-stream diagonal ratio among K users, i.e.,

$$\alpha^{*}\triangleq\min_{1\leq i\leq K}\alpha_{i}$$

- Connection with **Diagonal Dominance Matrix, DDM[2]**

$$\alpha^{*}\geq 1/2 \quad\Longrightarrow\quad |g_{ii}|>\sum_{j\neq i}^{K}|g_{ij}|,\forall i \quad\Longrightarrow\quad \mathbf{G}\ \text{is diagonally dominant matrix}$$

- Why does it mater?
  - ☐DDRI measures how close the XL-MIMO channel is to the **favorable** condition.
  - ☐A **larger** DDRI indicates a **stronger Gram-matrix diagonal dominance**
  - ☐DDRI can serve as a **direct decision metric** for **reliable low-complexity signal processing.**



Larger α^{*}

Stronger Diagonal Dominance

Favorable Channel Quality

Reliable Low-complexity Signal Processing

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[2] D. Zhu, B. Li, and P. Liang, "On the matrix inversion approximation based on Neumann series in massive MIMO systems," Proc. IEEE ICC, London, U.K., 2015, pp. 1763–1769.

03 Channel Quality Indicator: DDRI

- The **Effectiveness** of DDRI is **well-guaranteed** by the following two corollary

Corollary 1:

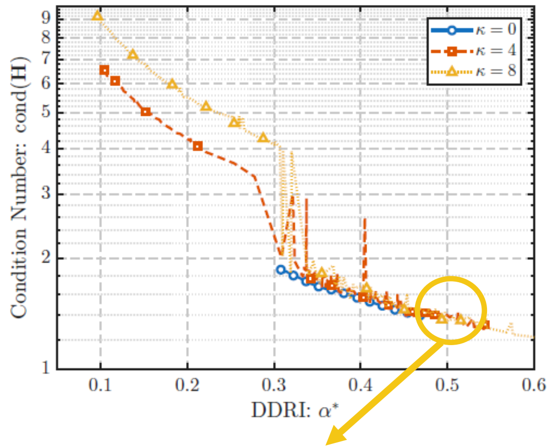
When $\alpha^* > 1/2$, the **condition number** of **H** is upper bounded by

$$\mathrm{cond}(\mathbf{H}) < \sqrt{g_{\max}/(g_{\min}(2\alpha^* - 1))}$$

Corollary 2:

Given $\bar{\alpha} = \mathrm{mean}_i\ \alpha_i$, the **channel rank** of **H** is lower bounded by

$$\mathrm{rank}(\mathbf{G}) > K\bar{\alpha}$$



The **Larger** α^*
The **Better** Channel Quality Is

- The framework of the proposed **DDRI-based Judgement Mechanism**

System Parameter Aquisition
$N, K, d_t, d_r, \theta_t, \theta_r$
✓ Near-field Positioning
✓ Near-field Channel Estimation

DDRI Computation

Channel Quality Evaluation
Threshold: $\alpha^* > 1/2$ or not
✓ Good → Low-complexity methods (i.e., CG)
✓ Poor → Traditional methods (i.e., MMSE, RZF)

Challenge 2: How to Compute DDRI Efficiently?

04 Efficient Approximation of DDRI

- The **Computational Efficiency** of DDRI is guaranteed by the following closed-form approximation.

Theorem 1: Closed-form Approximation of DDRI

Given the near-field channel matrix, for $\mathrm{N},\ \mathrm{K} \to \infty$, with $\mathrm{K}/\mathrm{N} < 1$, it follows that

$$\mathbb{E}[\overline{\alpha}] = \frac{\mathrm{N}}{\mathrm{N} + \frac{\kappa}{\kappa+1}\left(\frac{2}{\pi\eta}\ln(\mathrm{NK}\eta) + \mathrm{C}\right) + \frac{1}{\kappa+1}\left(\frac{\mathrm{K}-1}{2}\sqrt{\pi \mathrm{N}}\right)},$$

where $\boldsymbol{\eta = \pi d_t d_r cos\theta_t cos\theta_r/(\lambda r_0)}$ is **near-field coupling parameter** bearing the information of system configurations.

Highlights

- ✓ Enables **efficient** near-field channel quality evaluation with $\boldsymbol{\mathcal{O}(1)}$ complexity
- ✓ Jointly accounts for **LoS** and **NLoS** contributions, allowing adaptation to **different propagation scenarios** in XL-MIMO systems through the Rician K-factor

04 Sketch of Proof (Theorem 1)

- ☐ Decompose the Gram matrix into LoS and NLoS contributions.

Channel Model

$$\mathbf{H}=\sqrt{\frac{\kappa}{\kappa+1}}\mathbf{H}_{\mathrm{LoS}}+\sqrt{\frac{1}{\kappa+1}}\mathbf{H}_{\mathrm{NLoS}},$$

Gram Matrix Decomposition

$$\mathbf{G}=\frac{\kappa}{\kappa+1}\mathbf{H}_{\mathrm{LoS}}^{H}\mathbf{H}_{\mathrm{LoS}}+\frac{1}{\kappa+1}\mathbf{H}_{\mathrm{NLoS}}^{H}\mathbf{H}_{\mathrm{NLoS}}+\frac{\sqrt{\kappa}}{\kappa+1}(\mathbf{H}_{\mathrm{LoS}}^{H}\mathbf{H}_{\mathrm{NLoS}}+\mathbf{H}_{\mathrm{NLoS}}^{H}\mathbf{H}_{\mathrm{LoS}}).$$

Expectation

$$\mathbb{E}[\mathbf{G}]=\frac{\kappa}{\kappa+1}\mathbf{G}_{\mathrm{LoS}}+\frac{1}{\kappa+1}\mathbb{E}[\mathbf{G}_{\mathrm{NLoS}}].$$

- ☐ Approximate the **diagonal term**: both LoS and NLoS parts yield $\left|g_{ii}^{LoS}\right|=\tilde{\beta}^2 N,\left|g_{ii}^{NLoS}\right|=\tilde{\beta}^2 N$

- ☐ Approximate the **off-diagonal interference**
  - ➢ LoS contributions

LoS Channel Decomposition

$$\mathbf{G}_{\mathrm{LoS}}=\tilde{\beta}^2[\mathbf{\Phi}_r\mathbf{V}\mathbf{\Phi}_t^H]^H\mathbf{\Phi}_r\mathbf{V}\mathbf{\Phi}_t^H \overset{(a)}{=} \tilde{\beta}^2\mathbf{\Phi}_t\mathbf{V}^H\mathbf{V}\mathbf{\Phi}_t^H,$$

LoS Off-diagonal Elements

$$|g_{ij}^{\mathrm{LoS}}|=\tilde{\beta}^2\left|\frac{\sin(N\eta\delta)}{\sin(\eta\delta)}\right| \text{ for } i\neq j.$$

Lemma 1: LoS contributions

For $N,\ K\to\infty$, with $K/N<1$, by defining $G_i^{LoS}=\sum_{j\neq i}\left|g_{ij}^{LoS}\right|$, it follows that

$$G_i^{\mathrm{LoS}}\approx\frac{\tilde{\beta}^2}{\eta}\left(\frac{2}{\pi}\ln(NK\eta)+C\right),$$

where C is a constant independent of N, K and η.

  - ➢ NLoS contributions

NLoS Off-diagonal Elements

$$[\mathbf{H}_{\mathrm{NLoS}}]_{n,k}\sim\tilde{\beta}\mathcal{CN}(0,1).$$

$$|g_{ij}^{\mathrm{NLoS}}|=|(\mathbf{h}_i^{\mathrm{NLoS}})^{\mathrm{H}}\mathbf{h}_j^{\mathrm{NLoS}}|=\tilde{\beta}^2|\mathbf{z}_i^{\mathrm{H}}\mathbf{z}_j|.$$

Lemma 2: NLoS Contributions

For $N,\ K\to\infty$, with $K/N<1$, by defining $G_i^{NLoS}=\sum_{j\neq i}\left|g_{ij}^{NLoS}\right|$, it follows that

$$\mathbb{E}[G_i^{\mathrm{NLoS}}]\approx(K-1)\tilde{\beta}^2\frac{\sqrt{\pi N}}{2}.$$

Corollary 2: NLoS Contributions

For $N,\ K\to\infty$, with $K/N<1$, by defining $G_i^{NLoS}=\sum_{j\neq i}\left|g_{ij}^{NLoS}\right|$, it follows that.

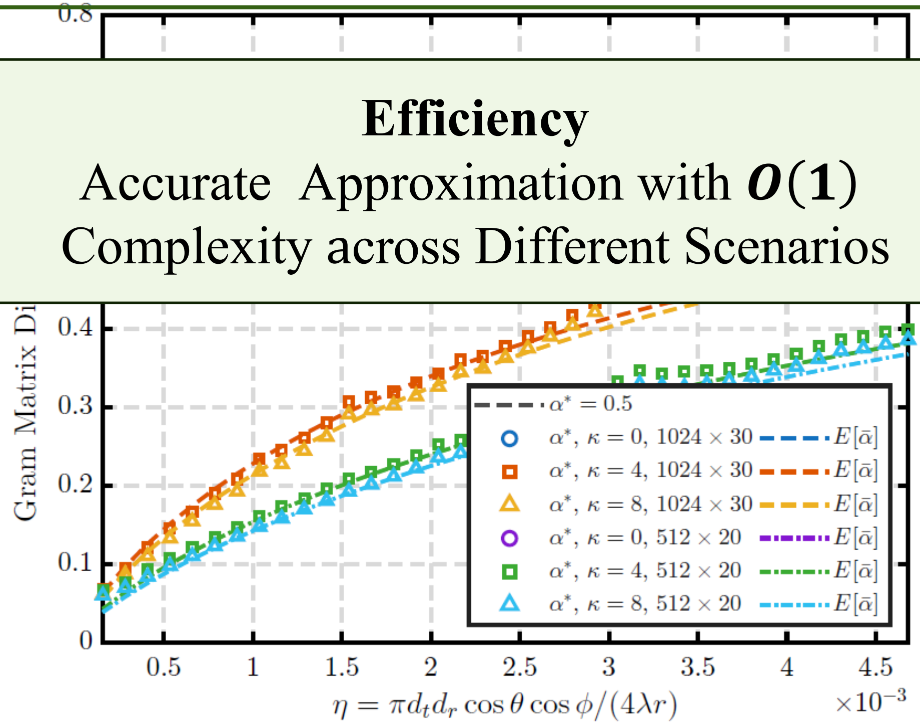
$$\mathbb{E}[G_i^{\mathrm{NLoS}}]\approx(K-1)\tilde{\beta}^2\frac{\sqrt{\pi N}}{2}.$$

- ☐ Substitute these **four terms** into the diagonal ratio and obtain an **$\mathcal{O}(1)$ closed-form approximation** of DDRI (**Efficiency**)

05 Numerical Results

Approximation Accuracy

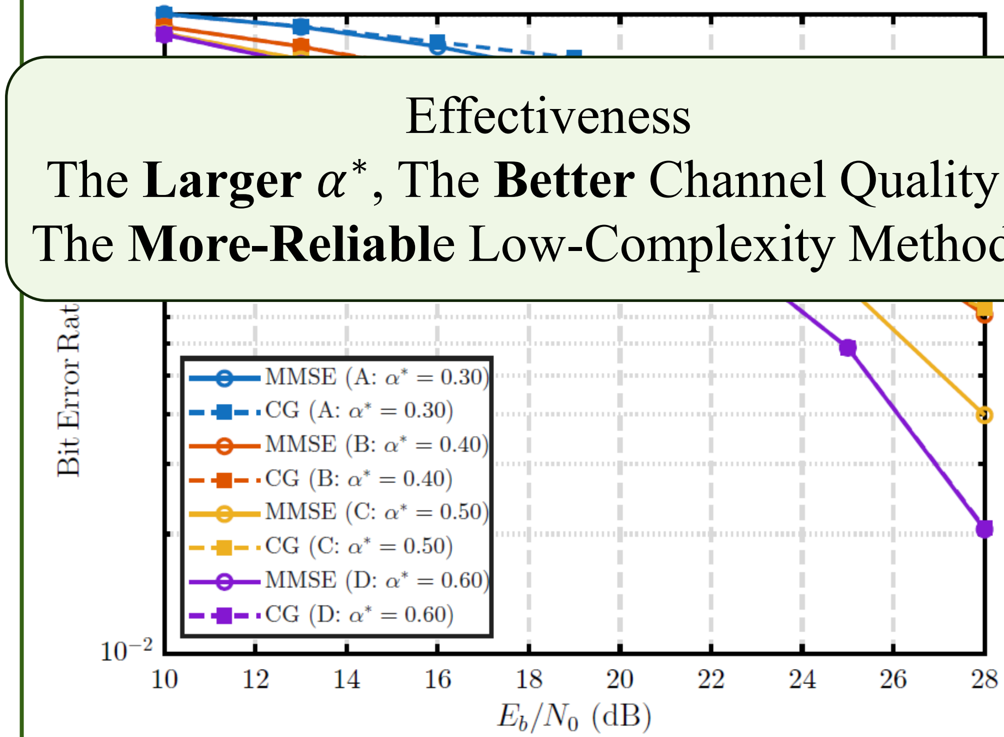
Efficiency
Accurate Approximation with $\boldsymbol{O}(\mathbf{1})$
Complexity across Different Scenarios



- The closed-form expression in Theorem 1 provides a accurate approximation of DDRI.
- The results demonstrated stable performance across large antenna sizes (1024×30, 512×20) and varying Rician ($\kappa = 0,\ ss4,\ 8$) parameters.

The Application of DDRI

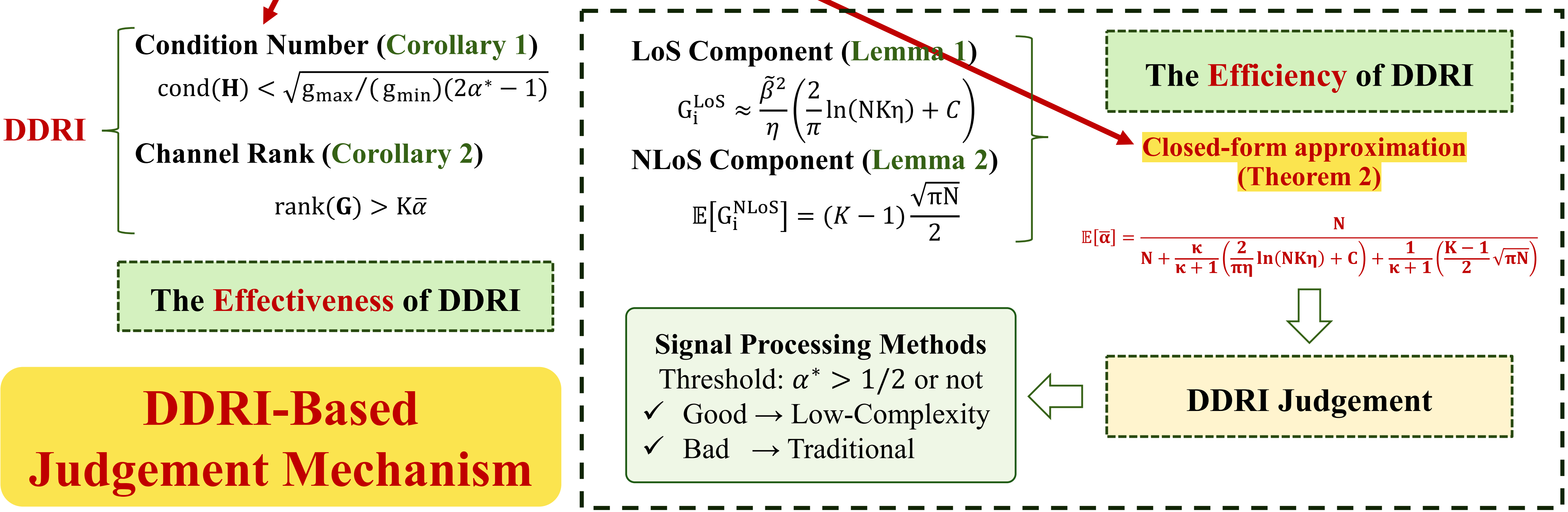
Effectiveness
The **Larger** α^*, The **Better** Channel Quality Is
The **More-Reliable** Low-Complexity Method Is



- The closed-form expression in Theorem 1 provides a good approximation of DDRI.
- The results demonstrated stable performance across large antenna sizes (1024×30, 512×20) and varying Rician ($\kappa = 0,\ 4,\ 8$) parameters.

Conclusion

We propose an effective and computationally efficient indicator to evaluate the channel quality of XL-MIMO systems, facilitating the design of low-complexity signal processing methods.



DDRI

Condition Number (Corollary 1)

$$\mathrm{cond}(\mathbf{H}) < \sqrt{g_{\max}/(g_{\min})(2\alpha^* - 1)}$$

Channel Rank (Corollary 2)

$$\mathrm{rank}(\mathbf{G}) > K\bar{\alpha}$$

The Effectiveness of DDRI

LoS Component (Lemma 1)

$$G_i^{\mathrm{LoS}} \approx \frac{\tilde{\beta}^2}{\eta}\left(\frac{2}{\pi}\ln(NK\eta) + C\right)$$

NLoS Component (Lemma 2)

$$\mathbb{E}\left[G_i^{\mathrm{NLoS}}\right] = (K-1)\frac{\sqrt{\pi N}}{2}$$

The Efficiency of DDRI

Closed-form approximation (Theorem 2)

$$\mathbb{E}[\bar{\alpha}] = \frac{N}{N + \frac{\kappa}{\kappa+1}\left(\frac{2}{\pi\eta}\ln(NK\eta) + C\right) + \frac{1}{\kappa+1}\left(\frac{K-1}{2}\sqrt{\pi N}\right)}$$

DDRI Judgement

Signal Processing Methods

Threshold: $\alpha^* > 1/2$ or not

✓ Good → Low-Complexity

✓ Bad → Traditional

DDRI-Based Judgement Mechanism

Thank you for your attention

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